



National Center  
on Response to  
Intervention

# ***RTI and Mathematics***

## **IES PRACTICE GUIDE - RESPONSE TO INTERVENTION IN MATHEMATICS**

---

Russell Gersten, Ph.D  
Director, Instructional Research Group  
Professor Emeritus, University of Oregon



U.S. Office of Special  
Education Programs



AMERICAN  
INSTITUTES  
FOR RESEARCH



# Panelists

---

- ❖ Russell Gersten (Chair)
- ❖ Sybilla Beckmann
- ❖ Ben Clarke
- ❖ Anne Foegen
- ❖ Laurel Marsh
- ❖ Jon R. Star
- ❖ Bradley Witzel



# Structure of the Practice Guide

---

- ❖ Recommendations
- ❖ How to carry out the recommendations
- ❖ Levels of Evidence
- ❖ Potential Roadblocks & Suggestions



# The Research Evidence

---

- ❖ The panel considered:
  - High quality experimental and quasi-experimental studies.
  - Also examined studies of screening and progress monitoring measures for recommendations relating to assessment.
- ❖ Qualitative and descriptive research served as background.



# Evidence Rating

---

- ❖ Each recommendation receives a rating based on the strength of the research evidence.
  - Strong
  - Moderate
  - Low



# Key Principles of RTI

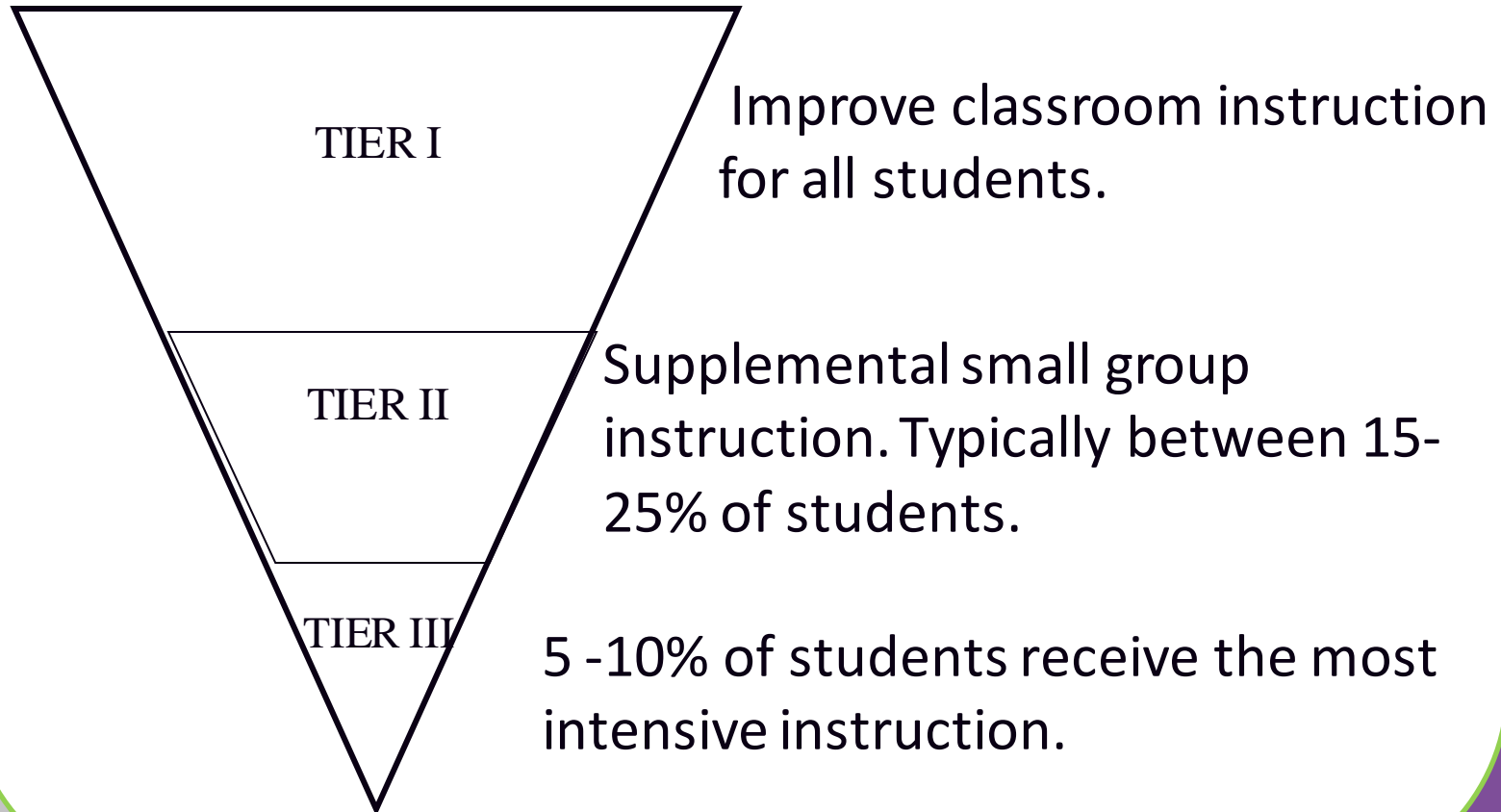
---

- ❖ Incorporate prevention and early intervention rather than waiting for failure.
- ❖ Include universal screening to identify student needs.
- ❖ Effective practices implemented class-wide in general education (primary intervention or Tier 1).
- ❖ Successive levels of support increasing in intensity and specificity provided to students as needed (secondary/tertiary intervention).



# RTI Model

---





<b>Recommendation</b>	<b>Level of Scientific Evidence</b>
1. Universal screening (Tier I)	Moderate
2. Focus instruction on whole number for grades k-5 and rational number for grades 6-8	Low
3. Systematic instruction	Strong
4. Solving word problems	Strong
5. Visual representations	Moderate
6. Building fluency with basic arithmetic facts	Moderate
7. Progress monitoring	Low
8. Use of motivational strategies	Low





# Recommendation 1

---

Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.

Level of Evidence: **Moderate**



# Evidence

---

- ❖ Technical evidence solid for early grades.
- ❖ Content of Measures
  - Single aspect of number sense (e.g. strategic counting, magnitude comparison) for K/1.
  - For grades 2 and up: Probably measures reflecting major state standards, National Mathematics Panel Benchmarks, Core Standards when they evolve etc. (*A lot of work to do here*)



# Examples of Missing Number Items

---

\_\_\_\_\_, 20, 21

8, \_\_\_\_\_, 10

9, 10, \_\_\_\_\_



# Magnitude Comparison

---

Which is bigger?

❖ 11 or 9?

❖ 79 or 95?

❖ 19 or 23?



# Roadblocks

---

- ❖ Screening may identify students as at-risk who do not need services and miss students who do.
- ❖ Suggested Approach: Consider delaying screening in kindergarten and first grade until November.



# Roadblocks

---

❖ Screening may identify large numbers of students who need support beyond the current resources of the school or district.

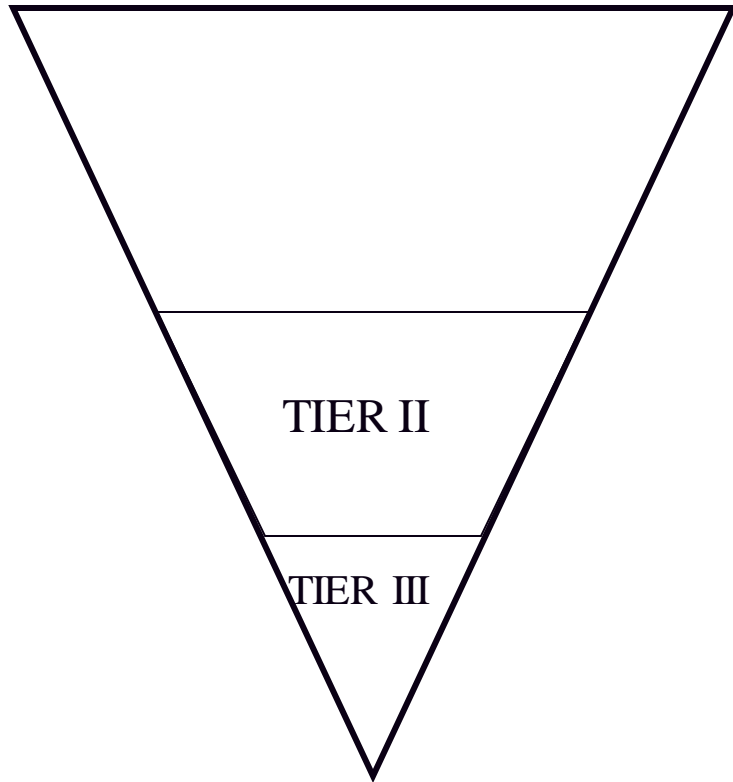
❖ Suggested Approach:

**Audience Suggestions**



# TIER II & TIER III

---



## Tier II

- Is individual or small-group intervention *in addition to the time allotted for core mathematics instruction.*
- Includes curriculum, strategies, and procedures designed to *supplement, enhance, and support* core classroom instruction.
- Can backtrack and/or elaborate/reinforce classroom curriculum.

## Tier III

- Includes some one-to-one work and more intense methods.



## Recommendation 2

# What to Teach in Intervention

---

Instructional materials for students receiving interventions should focus in-depth on:

- Whole numbers in kindergarten through grade 6
- Rational numbers in grades 4 through 8
- Applications to geometry and measurement

Level of Evidence: **Low**





# Evidence

---

- ❖ Consensus across mathematicians, professional organizations, and research panels
  - National Council Teachers of Mathematics (NCTM) and National Mathematics Advisory Panel (NMAP)
  - International comparisons
  - *We made the leap to nature of intervention curricula.....*



- 
- ❖ Instruction includes:
    - procedures
    - AND concepts
    - AND word problems
  - ❖ Whole number work consistently links operations to number properties



---

## Commutative Property

- $8 + 7 = 7 + 8$
- $A * b = b * a$

## Associative

- $a(bc) = (ab)c$
- $6 + (4 + 10) = (6 + 4) + 10$

## Distributive

- $7(13) = 7(10) + 7(3)$   
 $= 70 + 21$
- $a(b + c) = ab + ac$



# Fractions Defined

---

- ❖ Fractions arise naturally whenever we want to consider one or more parts of an object or quantity that is divided into pieces.
  - $\frac{1}{8}$  of a pizza
  - $\frac{2}{3}$  of the houses in the neighborhood
  - $\frac{3}{4}$  of a cup of water
- ❖ All of these examples use the word *of*, and all the fractions represent part *of* some object, collection of objects, or quantity.

Source: Beckmann, 2008 Mathematics for Elementary Teachers (2nd Ed.)
- ❖ Dilemma: how to convey this to kids
- ❖ Precursor: teacher must understand all this so that she or he can teach it



## Recommendation 3

---

Instruction during the intervention should be **systematic and explicit**. This includes providing models of proficient problem-solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

Level of Evidence: **Strong**



# Evidence

---

- ❖ Six randomized controlled trials met standards
- ❖ Key themes
  1. *Extensive practice with feedback*
  2. *Let students provide rationale for their decisions*
  3. *Instructors and fellow students model approaches to problem solving*



## Example

---

- ❖ Assignment: Use the lowest common denominator when appropriate

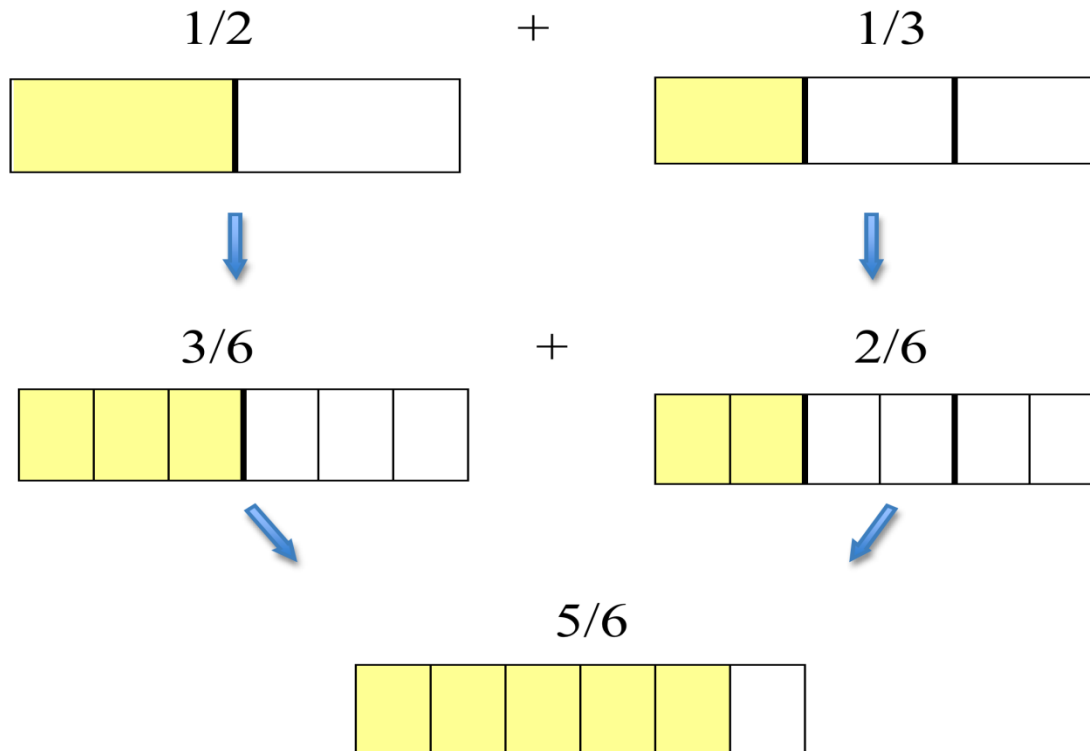
$$\frac{1}{2} + \frac{1}{3} =$$

- ❖ Student Response

$$\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$$



# Explicit Instruction Helps with Understanding of Fractions







# Roadblocks

---

- ❖ Intervention curricula may not have explicit instruction and may underestimate the amount of practice and review needed by tier 2 and 3 students.
  
- ❖ Suggested Approach:
  1. Develop guidebooks for school staff to adapt the lessons.
  2. Add new review problems.



## Recommendation 4

---

Interventions should include instruction on solving word problems that is based on common underlying structures.

Level of Evidence: **Strong**



# Suggestions

---

- ❖ Teach students about the structure of various problem types, **how to categorize problems**, and how to determine appropriate solutions.
  
- ❖ Middle step –
  - Is it:
    - Quantity (compare)
    - Change (over time)



## Compare Problems (Quantity)

---

There are 21 hamsters and 32 kittens at the pet store. How many more kittens are at the pet store than hamsters?





# Solving Similar Problems that Appear Different

---

- ❖ Difficulties encountered by some students
  - Extraneous information
  - Different wording
- ❖ Even though the problems have a common underlying structure
- ❖ Creates problems for any student who needs intervention



## Recommendation 5

---

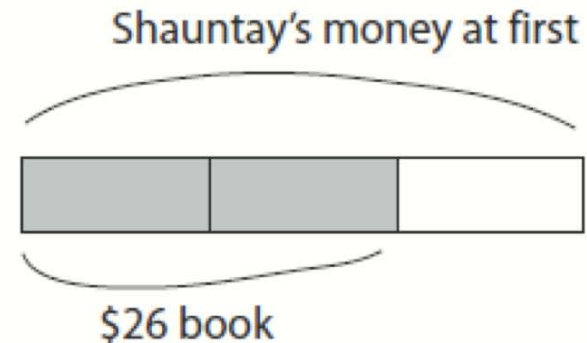
Intervention materials should include opportunities for the student to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

Level of Evidence: **Moderate**



# Strip Diagrams Can Help Students Make Sense of Fractions

Shauntay spent  $\frac{2}{3}$  of the money she had on a book that cost \$26. How much money did Shauntay have before she bought the book?



2 parts  $\longrightarrow$  \$26

1 part  $\longrightarrow$   $\$26 \div 2 = \$13$

3 parts  $\longrightarrow$   $3 \times \$13 = \$39$

Shauntay had \$39



# Suggestions

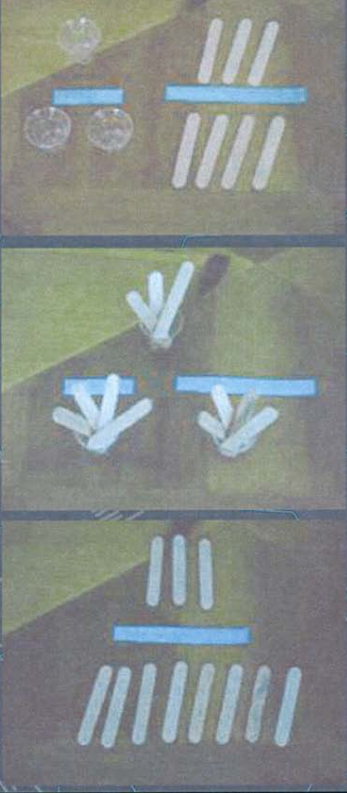
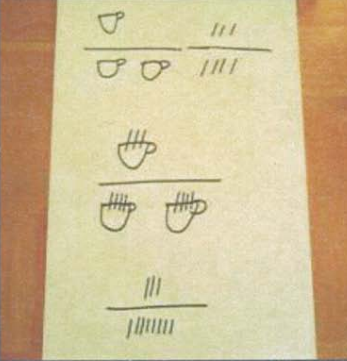
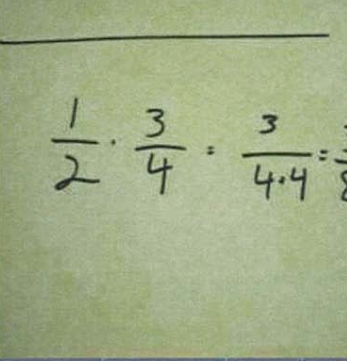
---

- ❖ Use visual representations such as number lines, arrays, and strip diagrams.
- ❖ If necessary consider expeditious use of concrete manipulatives before visual representations. The goal should be to move toward abstract understanding.





National Center  
on Response to  
Intervention

Concrete	Representational	Abstract
 <p>The concrete representation shows three stages of a multiplication problem. In the first stage, two blocks represent 1/2 and three sticks represent 3/4. In the second stage, the blocks are rearranged to show the product. In the third stage, the product is shown as a single block representing 3/8.</p>	 <p>The representational representation shows three stages of a multiplication problem. In the first stage, two hand-drawn symbols represent 1/2 and three hand-drawn symbols represent 3/4. In the second stage, the symbols are rearranged to show the product. In the third stage, the product is shown as a single hand-drawn symbol representing 3/8.</p>	 <p>The abstract representation shows the multiplication problem written in mathematical notation: <math>\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{4 \cdot 4} = \frac{3}{8}</math>.</p>

Source: Witzel, 2008



## Recommendation 6

---

Interventions at all grades should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

Level of Evidence: **Moderate**



# Suggestions

---

- ❖ Provide 10 minutes per session of instruction to build quick retrieval of basic facts.
- ❖ For student in K-2 grade explicitly teach strategies for efficient counting to improve the retrieval of math facts.
- ❖ Teach students in grades 2-8 how to use their knowledge of math properties to derive facts in their heads.



## Recommendation 7

---

Monitor the progress of students receiving supplemental instruction and other students who are at risk.

Level of evidence: **Low**



# Evidence

---

- ❖ Non-experimental studies demonstrating the technical adequacy of progress monitoring measures.
- ❖ General outcome measures reflecting concepts and computation objectives for the grade level.
- ❖ Greater evidence in elementary grades.



# Suggestions

---

- ❖ Monitor the progress of tier 2, tier 3 and borderline tier 1 students at least once a month using grade appropriate general outcome measures.
  
- ❖ Use curriculum-embedded assessments in intervention materials
  - Frequency of measures can vary - every day to once every week.



## Recommendation 8

---

Include motivational strategies in tier 2 and tier 3 interventions.

Level of Evidence: **Low**



# Roadblocks

---

- ❖ Rewards can reduce genuine interest in mathematics by directing student attention to gathering rewards rather than learning math.
- ❖ Suggested Approach: Rewards have not shown to reduce intrinsic interest. As students become more successful rewards can be faded so student success becomes an intrinsic reward.





National Center  
on Response to  
Intervention

---

# Questions?



# Resources

---

- ❖ Center on Instruction (COI)  
[http://www.centeroninstruction.org/resources.cfm?category=math&subcategory=&grade\\_start=&grade\\_end=#226](http://www.centeroninstruction.org/resources.cfm?category=math&subcategory=&grade_start=&grade_end=#226)
- ❖ National Center for Learning Disabilities (NCLD) RTI Action Network <http://www.rtinetwork.org/>
- ❖ Glover, T. A., & Vaughn, S. (in press). The promise of response to intervention: Evaluating current science and practice. New York: Guilford Press.
- ❖ WWC Practice Guide  
<http://ies.ed.gov/ncee/wwc/publications/practiceguides/>



National Center  
on Response to  
Intervention

---

Thank you

Contact Information:

<http://www.inresg.org/>